This Mathematica Notebook contains computations and derivations used to create plots in Figs. 8, 9 and 10 of our paper Ohzawa, DeAngelis, Freeman, Encoding of Binocular Disparity by Complex Cells in the Cat's Visual Cortex, J. Neurophysiol. 77: 2879-2909, (1997).

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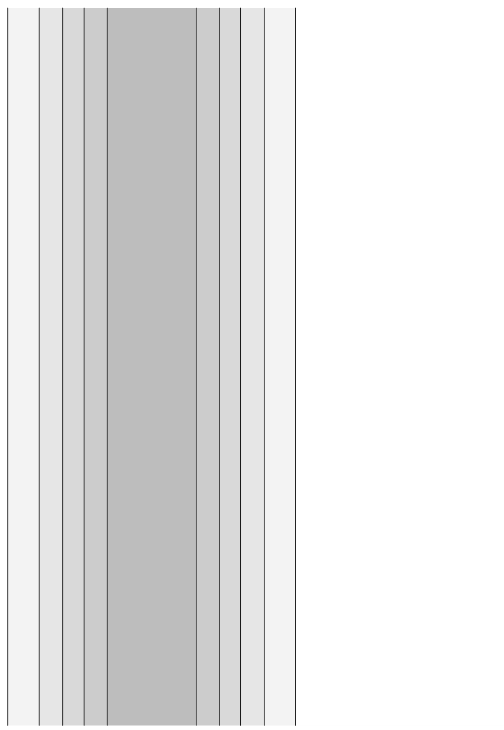
Monocular response (Left eye) of Cx cell based on the energy model is
Lresp (xL) =
(Exp[-k xL^2] Cos[2 Pi xL])^2 + (Exp[-k xL^2] Sin[2 Pi xL])^2

= (Exp[-k xL^2])^2
= Exp[-2 k xL^2]

k is proportional to the inverse of subunit RF width, and
pD is the phase difference of L and R subunit RF fields
(same for all 4 subunits).

Monocular response for the Left eye is given by,

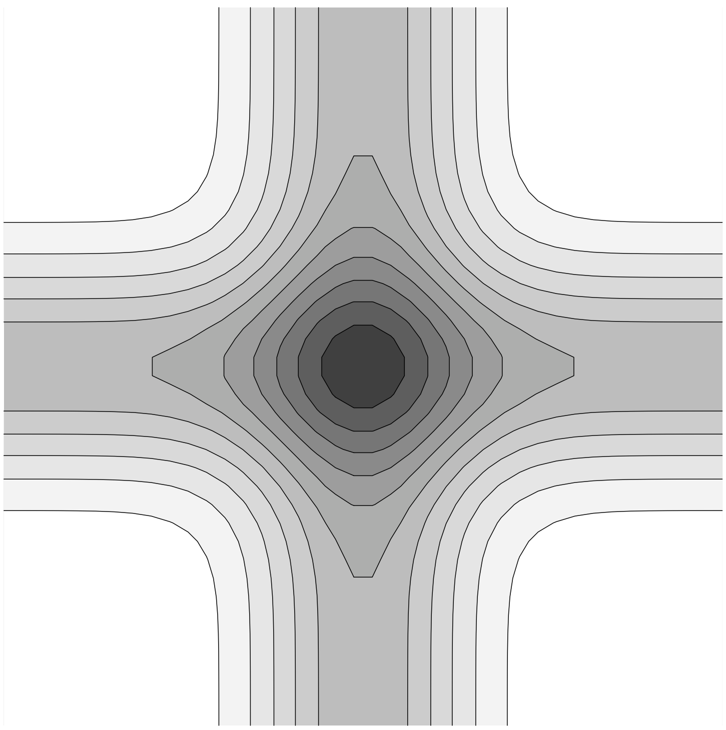
```
z[x_]:=GrayLevel[1-x];
pD:= 0;
k:=5.5;
ContourPlot[
Exp[- 2 k xL^2]
,
   {xL, -1, 1}, {xR, -1, 1},
   Contours -> 12, PlotRange -> {0, 2.2},
   ColorFunction -> z,
   PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

Sum of Left and Right monocular responses (which may be a good model for a non-phase specific cells) is given by:

```
z[x_]:=GrayLevel[1-x];
pD:= 0;
k:=5.5;
ContourPlot[
( Exp[- 2 k xL^2]  + Exp[-2 k xR^2] )
,
   {xL, -1, 1}, {xR, -1, 1},
   Contours -> 12, PlotRange -> {0, 2.2},
   ColorFunction -> z,
   PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

This is the derivation of Equation **7** from Equation **5** in our paper
Ohzawa, DeAngelis, Freeman,
Encoding of Binocular Disparity by Complex Cells in the Cat's Visual Cortex,
J. Neurophysiol. 77: 2879-2909, (1997).

Binocular response for matched contrast conditions (BB and DD)
may be simplified as follows:

```
Simplify[
( Exp[-k xL^2] Cos[2 Pi F xL] +Exp[-k xR^2] Cos[2 Pi F xR + pD])^2
+(Exp[-k xL^2] Sin[2 Pi F xL] +Exp[-k xR^2] Sin[2 Pi F xR + pD])^2
]
```

$$E^{-2\ k\ xL^2} + E^{-2\ k\ xR^2} + \frac{2\ \mathrm{Cos}[pD - 2\ F\ Pi\ xL + 2\ F\ Pi\ xR]}{E^{k\ (xL^2 + xR^2)}}$$

Just to be general, try the case where spatial freq are different between L and R eyes (use FL and FR for each eye).

```
Simplify[
(Exp[-k xL^2] Cos[2 Pi FL xL]
    + Exp[-k xR^2] Cos[2 Pi FR xR + pD])^2
+(Exp[-k xL^2] Sin[2 Pi FL xL]
    + Exp[-k xR^2] Sin[2 Pi FR xR + pD])^2
]
```

$$E^{-2\ k\ xL^2} + E^{-2\ k\ xR^2} + \frac{2\ \mathrm{Cos}[pD - 2\ FL\ Pi\ xL + 2\ FR\ Pi\ xR]}{E^{k\ (xL^2 + xR^2)}}$$

As shown above, the first two terms are monocular responses.
The last term is the binocular interaction component, which is a Gabor function (Is this really a Gabor?)
that is oriented at 45 degs in (xL, xR)
domain (see below).

The binocular component alone may be obtained without separate monoculuar measurements by computing BB+DD-BD-DB. Since BB == DD and BD == DB, because of squaring which makes the function independent of the inversion of sign,

BB+DD-BD-DB = 2 (BB-BD)

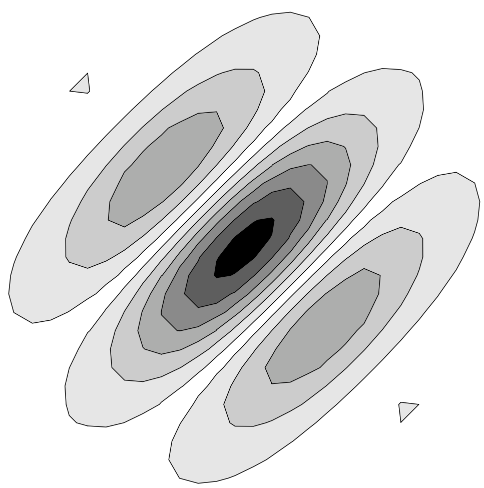
BB-BD is given by below which is reduced the same expression as above.

```
k=.;
pD=.;
Simplify[
(Exp[-k xL^2] Cos[2 Pi xL] +Exp[-k (xR)^2] Cos[2 Pi (xR) + pD])^2
+(Exp[-k xL^2] Sin[2 Pi xL] +Exp[-k (xR)^2] Sin[2 Pi (xR) + pD])^2
- (
(Exp[-k xL^2] Cos[2 Pi xL] -Exp[-k (xR)^2] Cos[2 Pi (xR) + pD])^2
+(Exp[-k xL^2] Sin[2 Pi xL] -Exp[-k (xR)^2] Sin[2 Pi (xR) + pD])^2
) ]
```

$$\frac{4\ \mathrm{Cos}[pD - 2\ Pi\ xL + 2\ Pi\ xR]}{E^{k\ (xL^2 + xR^2)}}$$

This is a Gabor function oriented at 45 degs in the (xL, xR) domain
as shown below: (ModelDecomp/GaborLR-0.eps)

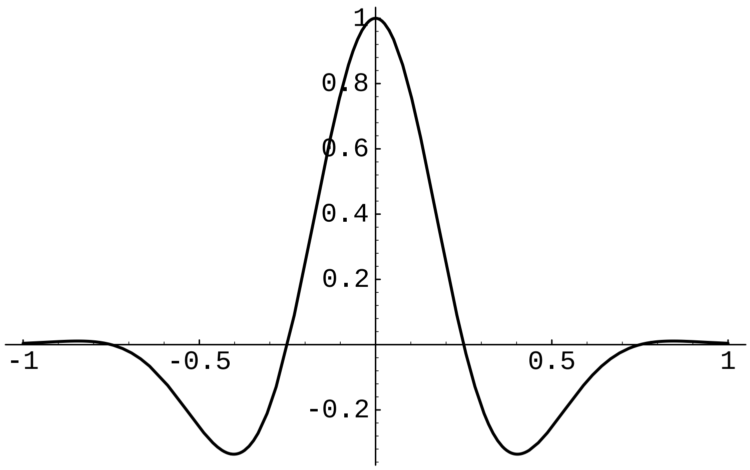
```
z[x_]:=GrayLevel[1-2*Abs[x-0.5]];
pD:= 0;
k:=5.5;
ContourPlot[
  Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {-1.1, 1.1},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

k:=5.5;

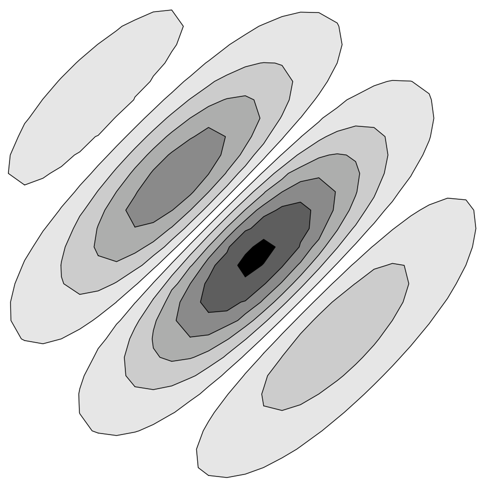
```
Plot[
  Exp[-k (xL^2)] Cos[2 Pi xL],
     {xL, -1, 1},
     PlotPoints -> 40 ]
```



-Graphics-

Phase Difference = 45 degs (ModelDecomp/GaborLR-45.eps)

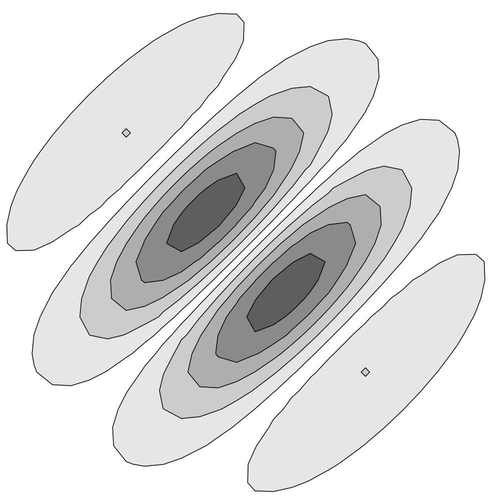
```
z[x_]:=GrayLevel[1-2*Abs[x-0.5]];
pD:= Pi/4;
k:=5.5;
ContourPlot[
  Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {-1.1, 1.1},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

Phase Difference = 90 degs (ModelDecomp/GaborLR-90.eps)

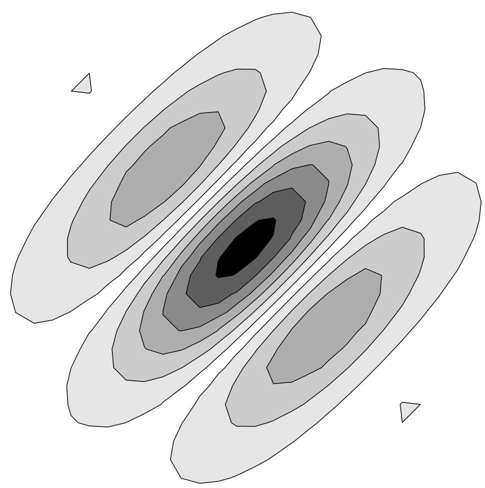
```
z[x_]:=GrayLevel[1-2*Abs[x-0.5]];
pD:= Pi/2;
k:=5.5;
ContourPlot[
  Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {-1.1, 1.1},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

Phase Difference = 180 degs (ModelDecomp/GaborLR-180.eps)

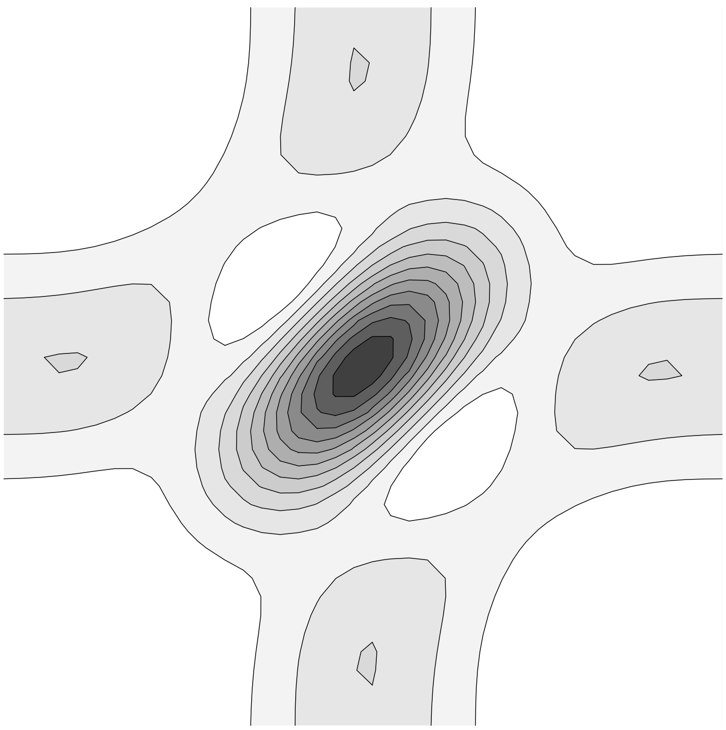
```
z[x_]:=GrayLevel[1-2*Abs[x-0.5]];
pD:= Pi;
k:=5.5;
ContourPlot[
  Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {-1.1, 1.1},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

Zero disparity detector for both phase and position models is the same:

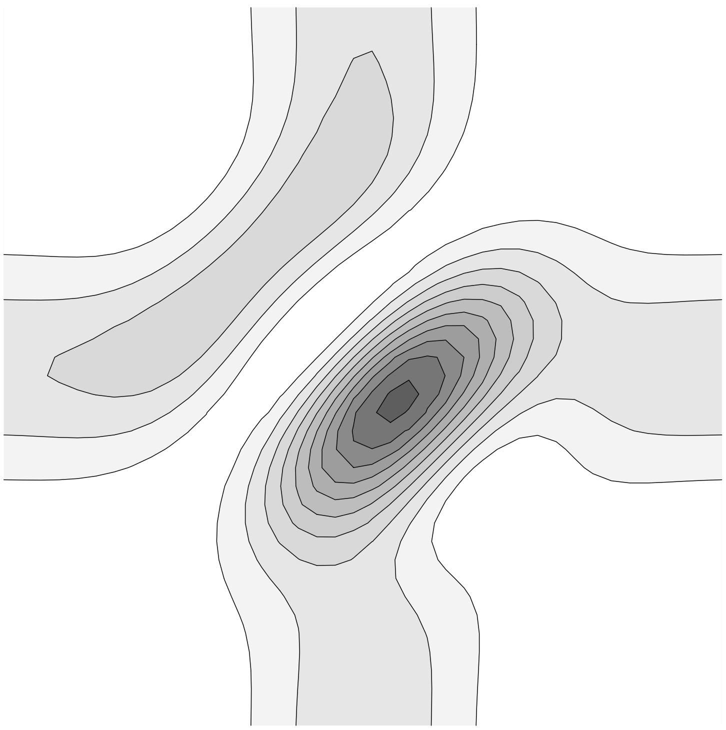
```
z[x_]:=GrayLevel[1-x];
pD:= 0;
k:=5.5;
ContourPlot[
    Exp[- 2 k xL^2]  + Exp[-2 k xR^2]
  + 2 Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {0, 4.4},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

Non-Zero disparity detector for the phase model

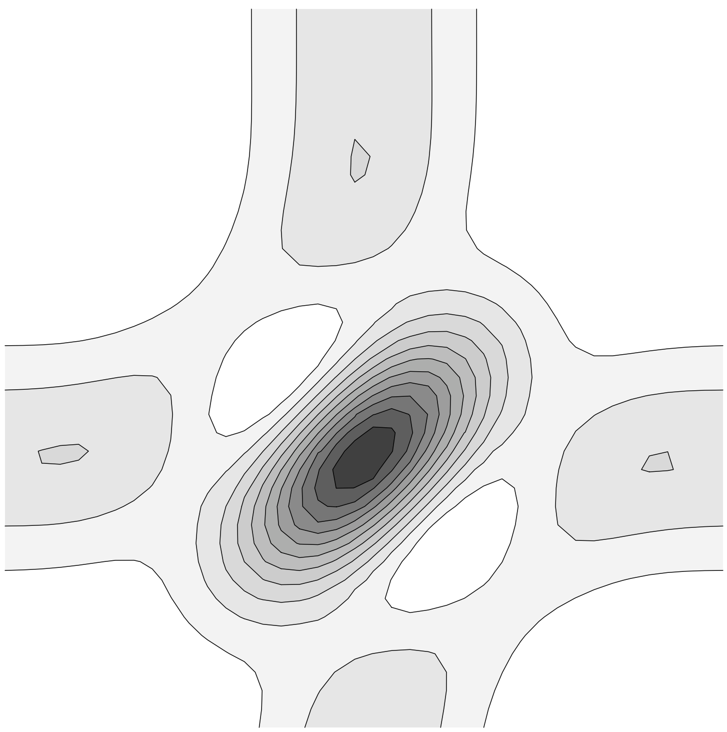
```
z[x_]:=GrayLevel[1-x];
pD:= Pi/2;
k:=5.5;
ContourPlot[
    Exp[- 2 k xL^2]  + Exp[-2 k xR^2]
  + 2 Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {0, 4.4},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

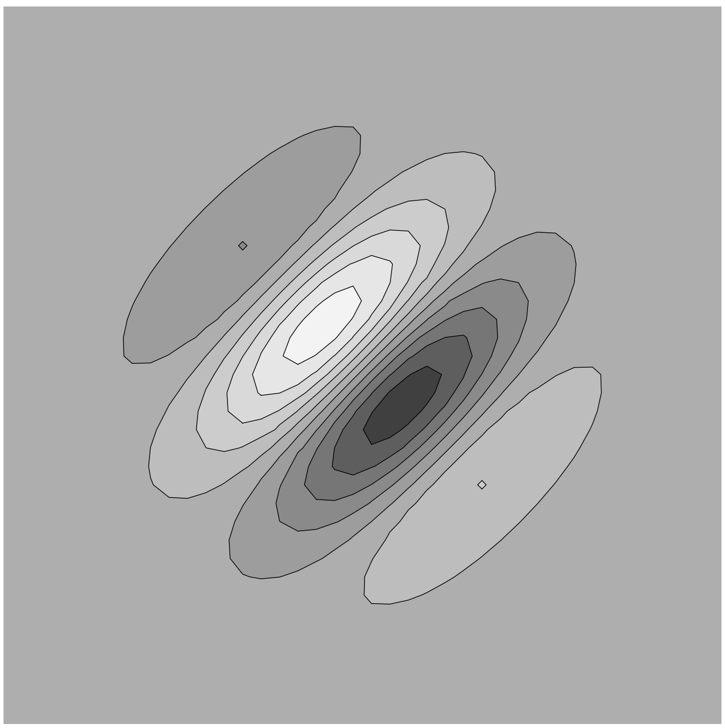
Non-Zero disparity dector for the Position model

```
z[x_]:=GrayLevel[1-x];
oR:= 0.25;
pD:= 0;
k:=5.5;
ContourPlot[
    Exp[- 2 k xL^2]  + Exp[-2 k (xR+oR)^2]
  + 2 Exp[-k (xL^2 + (xR+oR)^2)] Cos[2 Pi (xL - (xR+oR)) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {0, 4.4},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



```
-ContourGraphics-
```

```
z[x_]:=GrayLevel[1-x];
pD:= Pi/2;
k:=5.5;
ContourPlot[
  Exp[-k (xL^2 + xR^2)] Cos[2 Pi (xL - xR) - pD],
     {xL, -1, 1}, {xR, -1, 1},
     Contours -> 12, PlotRange -> {-1.1, 1.1},
     ColorFunction -> z,
     PlotPoints -> 40,  Axes -> None, Frame -> False ]
```



-ContourGraphics-